# Aeromagnetic 3D subsurface imaging with source volume minimization

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# ABSTRACT

We have been developing an integrated system for helicopter-borne magnetic survey, from the measuring equipment and data processing to data reduction procedures and interpretation. In this paper, we deal with the effective method of 3D subsurface imaging from the helicopter-borne aeromagnetic anomaly data in a mountainous region. Based on the works by Li and Oldenburg (1996), Pilkington (1997), Portniaguine and Zhdanov (2002), etc., we discussed the simple minimum norm solution of 3D subsurface imaging, and pointed out the importance of the parameter weighting related to the volumes of source models (Nakatsuka and Okuma, 2006c). However, the simple minimum norm solution cannot give a focused image of subsurface structure, but tends to result in a broader shape of source with vague boundary especially towards deeper side as an equivalent source model. It is expected that we can overcome this point by the introduction of the compactness criterion (Last and Kubik, 1983; Portniaguine and Zhdanov, 2002). In treating the actual magnetic anomaly data, it is obvious that the resolving power for the subsurface structure depends on its depth, and the modeling with variable source size (coarser at deeper sources) is preferable. Portniaguine and Zhdanov (2002) used a method of minimizing the number of discrete sources, which is actually minimizing source volume, because they discussed the model consisting of constant volume cubes. If we use an analysis model with the ensemble of variable volume blocks, the scheme of minimizing source volume is a little different. We are developing the analysis scheme for such model.

**KEY WORDS:** aeromagnetic survey, helicopter survey, mountainous region, 3D imaging, compact inversion, volume minimization

## INTRODUCTION

Recent developments in aeromagnetic surveying, including improved accuracy of position fixing and navigation, have enabled us to detect mode details of subsurface structure. We are developing an integrated system for helicopter-borne magnetic survey, from the measuring equipment and data processing to data reduction procedures and interpretation.

After the development of our survey equipment and the procedure to compensate aircraft magnetic field noise (Nakatsuka and Okuma, 2005), we treated the altitude reduction technique. Magnetic survey flights by helicopters are usually parallel to the topographic surface, but especially in high-resolution surveys the altitudes at which observations are made may be too variable to be regarded as a smooth surface. The procedure in use of the equivalent source technique was proven to give efficient way of altitude reduction (Nakatsuka and Okuma, 2006a).

Crossover analysis is an effective method to estimate the systematic error in the observation and is utilized for mis-tie adjustments. This mis-tie control procedure has been widely used for aeromagnetic surveys of constant barometric altitudes flights. In a helicopter survey of mountainous area, even at the intersection of horizontal trajectories of traverse and tie lines, the altitudes of both lines differ considerably in practice. This means that the conventional crossover analysis cannot be applied. As the present positional data acquired from the GPS system are quite reliable, we developed an algorithm of estimating the magnetic level shifts as an extension of the altitude reduction procedure using equivalent sources (Nakatsuka and Okuma, 2006b), and called it the "generalized mis-tie control method".

There is another application of this method in detecting the magnetic anomaly change by repeated surveys. Since the track lines of different surveys cannot be the same, there is a problem of spatial alias effect in comparing two datasets. In the method of generalized mis-tie control, the differences between the two surveys are evaluated only on the crossover (control) points and the method is useful to avoid the spatial alias effect. This method was successfully applied to actual data of Asama Volcano to depict magnetic anomaly change associated with the 2004 volcanic activity (Nakatsuka et al., 2009).

As for the interpretation of magnetic anomalies in terms of subsurface magnetic structure, we often used the method of magnetization intensity (2D) mapping (Okuma et al., 1994; Nakatsuka, 1995; Okubo et al., 2005). As is well known, the magnetic (potential field) anomaly data cannot give a unique solution of the subsurface structure theoretically, and there are infinite possibilities of equivalent 3D sources. So. some constraint on the source distribution is necessary to get valid result, and the result should be inspected for its plausibility. Unless another kind of observation is available for the unified interpretation of multi discipline data, some philosophical strategy should be given to overcome the difficulty.

The earth science can be regarded as the knowledge of the nature by simplifying the complicated situation and the diversity. The minimum norm solution (regularization) will be one candidate, and was effective for underdetermined solution of the magnetization intensity mapping. In the simple synthetic study of 3D imaging, however, it was proven that minimum norm criterion cannot give a focused image of subsurface structure, but tends to result in a broader shape of source with vague boundary especially towards deeper side as an equivalent source model (Nakatsuka and Okuma, 2006c). The selection of smoothness (minimum roughness) regularization, as often used in non-linear problem such as in EM methods (e.g., Uchida, 1993), seems to have similar character. Now we try to adopt the compactness criterion to get a localized source image from the observation, as was developed by Last and Kubik (1983), and Portniaguine and Zhdanov (2002).

Here we deal with the total force magnetic anomaly field. Although the total force magnetic field is not harmonic in a strict sense, it can be well approximated as the (harmonic) component in the normal field direction even in the cases of magnetic anomaly intensity more than 1000 nT. The difference between the two is marked only where total force anomaly is nearly zero, and the error of approximation does not affect the magnetic anomaly peaks, which are the essential information in the mapping.

### **ANALYSIS MODEL FOR 3D IMAGING**

As we are aiming for the application of 3D imaging to aeromagnetic surveys in mountainous regions such as active volcanoes, our model configuration has a surface undulation and with a variable depth slicing as the vertical section view is shown in Fig. 1. There are two options to define model configuration, (a) horizontal layer slicing, or (b) constant thickness slicing (variable layer by layer) from the surface. Here we introduced the additional source zone around the area of observation to avoid edge effect (Nakatsuka and Okuma, 2006a). As the analysis result in this zone of surrounding sources (away from observation) is not very reliable, usually we exclude them in the result illustration.

#### **3D IMAGING**

The basic formula of 3D subsurface imaging is completely the same as the equivalent source analysis or the magnetization intensity mapping, but the sources are distributed in three-dimensional space and have the physical quantity of magnetization. The magnetic anomaly f(p) at the observation point p is expressed as

$$f(\boldsymbol{p}) = \int_{\Omega} A(\boldsymbol{p}, \boldsymbol{q}) \, s(\boldsymbol{q}) \, ds \,, \tag{1}$$

where s(q) is the magnetization of the source located at q, and A(p, q) is the source effect coefficient (geometric factor) combining f(p) with s(q). Here the formula of magnetic anomaly of prism-shaped source bodies (e.g., Bhattacharyya, 1964) is used to calculate A(p, q), or it may be alternated with the formula of point source model if the mesh of source distribution is fine enough.

Nakatsuka and Okuma (2006c) solved the equation (1) by the conjugate gradient (CG) method. The CG method applied to underdetermined linear problem gives a minimum norm solution, and they discussed the problem of parameter scaling, which defines what kind of norm is to be minimized. However, the result from the simple CG solution could not give a focused image of subsurface structure.



Figure 1. Two types of layer configuration (section view) for 3D magnetization imaging software

As a solution for such problem, various methods of regularization are utilized. The generalized formulation for them is given as follows.

Using a vector/matrix notation, discretized form of the original equation (1) can be expressed as

$$f = \mathbf{A}\mathbf{s} \,, \tag{2}$$

and the simple CG method searches for the condition

$$(f - As)^{\perp}(f - As) \rightarrow Min$$
, (3)  
which corresponds to the normal equation for (2),

$$\mathbf{A}^{\mathrm{T}} \boldsymbol{f} = \mathbf{A}^{\mathrm{T}} \mathbf{A} \boldsymbol{s} \quad . \tag{4}$$

In case of the regularization methods, another term is added in the minimizing functional as

 $(f-As)^{T}(f-As) + e(Bs)^{T}(Bs) \rightarrow Min,$  (5) where **B** is the penalizing operator matrix, and *e* is a trade-off parameter (so-called hyper-parameter) to adjust the penalizing weight ratio between the former misfit term  $(f-As)^{T}(f-As)$  and the latter regularization term  $(Bs)^{T}(Bs)$  in (5). This formulation (5) corresponds to the normal equation for the combined equation

and the application of simple CG method to this equation is reduced to the regularized solution for the equation (2).

If a unit matrix is selected for **B**, the formulation (5) means the minimum norm regularization. Similarly, if **B** is Laplacian (roughness operator) matrix, the problem is reduced to the smooth inversion.

### **COMPACTNESS REGULARIZATION**

Last and Kubik (1983) first introduced a compact inversion technique in a 2-D gravity analysis, and Portniaguine and Zhdanov (2002) developed it in 3-D magnetic imaging. The idea is to adopt the penalizing operator matrix as

$$\mathbf{B} = \operatorname{diag}\left\{1/\sqrt{s_i^2 + \delta^2}\right\},\tag{7}$$

and the regularizing factor R of (5) becomes

$$R = (\mathbf{B}s)^{\mathrm{T}}(\mathbf{B}s) = \sum_{i} \frac{s_{i}^{2}}{s_{i}^{2} + \delta^{2}}$$
 (8)

Here  $\delta(>0)$  is a critical value of magnetization, and source bodies with small magnetization well below  $\delta$  $(|s_i| \ll \delta)$  are regarded as ineffective to buildup anomalies. As each element of *R* has a value between 0  $(|s_i| \ll \delta)$  and 1  $(|s_i| >> \delta)$ , the minimization of *R* corresponds to the minimization of number of sources contributing the anomaly buildup. This is equivalent to minimizing source volume if source model is composed



Figure 2. Constant volume and variable volume source body configurations (section view) of test analysis for point source model anomaly

of equi-volume bodies. In case of the model composed of variable volume bodies, as we want to use, the penalizing operator and the regularizing factor should be

$$\mathbf{B} = \operatorname{diag}\left\{\sqrt{\frac{v_i}{s_i^2 + \delta^2}}\right\},\tag{9}$$

$$R = (\mathbf{B}s)^{\mathrm{T}}(\mathbf{B}s) = \sum_{i} \frac{v_{i} s_{i}^{2}}{s_{i}^{2} + \delta^{2}},$$
 (10)

where  $v_i$  is the volume of *i*-th source body.

# **CHARACTER OF EACH METHOD**

In order to know the characteristic behavior of the above-mentioned regularization methods, analyses for a point (or sphere) source model data were performed. There are two types of layer configurations, constant and variable volume sources, and both consist of  $61 \times 61 \times 10$  source blocks including surrounding zone as in Fig. 2. Although the second configuration is not realistic in actual data analysis, it is only to know characteristic behavior.

Figure 3 shows the results for the constant volume source body configuration: (a) no depth scaling without any regularization, (b) depth scaled but without regularization, (c) minimum norm regularization, and (d) compactness regularization. The necessity of depth scaling is obvious, and it is proven that no regularization is actually equivalent to the minimum norm regularization. The result of compactness regularization (minimizing number of effective sources) gave very good image for the subsurface structure.

Figure 4 shows the results for the variable volume source body configuration: (a) minimum norm regularization without volume scaling, (b) regularization of minimizing number of effective sources, and (c) regularization of minimizing total volume of effective sources. It is obvious that the scaling for the variable volume of source body is essential to get reasonable result. The volume scaling for compactness regularization is easily implemented by using equations (9) and (10) in place of equations (7) and (8).

## MODEL EXAMPLES

Next the algorithm of minimum effective volume regularization was applied to synthetic model examples. In Fig. 5, there are three models nearly same as those discussed by Portniaguine and Zhdanov (2002), and



**Figure 3.** Effect of depth scaling and regularization [for constant volume source configuration]. The distribution of magnetization on the central NS vertical section, calculated with (a) no depth scaling/regularization, (b) only depth scaling, (c) minimum norm regularization, and (d) compactness regularization.





 $71 \times 71 \times 25$  source blocks including surrounding zone were assumed. The result is generally similar to theirs. Though the horizontal location is recovered well, the recovery of vertical structure is not so correct. This might indicates the limitation from the inherent non-uniqueness. Nevertheless the results of compact inversion methods can be informative enough to interpret magnetic anomalies in terms of 3D structure.

A field example applied to actual helicopter-borne survey data will also be presented at the Symposium.

#### DISCUSSIONS

In the process of iterative solution of the problem, the trade-off parameters e plays an important role. To minimize misfit term, e is decreased during the process. However, if e is decreased too fast, the existence of regularization term becomes ineffective and the solution resembles the minimum norm one. To make sure the solution is compact, relatively large value of e should be maintained as long as possible.



**Figure 5.** Application of minimum effective volume regularization for (a) cube model, (b) dipping slab model, and (c) faulted dipping slab model, as similar to those discussed by Portniaguine and Zhdanov (2002). Left: sketch of source body at the E-W vertical section; Center: synthetic magnetic anomaly (source data of analysis) with horizontal outline of source body; Right: result of compact inversion (imaging by minimum effective volume regularization) at the E-W vertical section at the center.

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