Discussion: On 'Magnetization structure of Aogashima Island using vector magnetic anomalies obtained by a helicopter-borne magnetometer' (Isezaki, N., and J. Matsuo, 2009, *Exploration Geophysics*, 40, 17–26; *Butsuri-Tansa*, 62, 17–26; *Mulli-Tamsa*, 12, 17–26).

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In 'Magnetization structure of Aogashima Island using vector magnetic anomalies obtained by a helicopter-borne magnetometer', Isezaki and Matsuo discussed a source of inaccuracy in the inversion of total intensity magnetic anomalies, and presented an example of a helicopter three-component magnetic survey and its inversion. In their paper, I consider there to be some problems and points to be discussed. Here I refer to the figures in their paper by the original figure numbers, and use the label D-1 for the new figure in this discussion.

Isezaki and Matsuo described the algebra of the total intensity anomaly (*TIA*), and discussed the error introduced by the assumption that *TIA* is approximated by the projected total intensity anomaly (*PTA*). Their figures 3 and 4 are correct although the description of the horizontal axis of figure 4 could be clearer. It is obvious that the relative error ε_T/TIA (=(*TIA* – *PTA*)/*TIA*) becomes infinite where *TIA* = 0. However, the situation is not as severe as their assertion implies, because such a large error occurs only in a quite limited region near *PTA* = 0, but the result of analyses of magnetic anomalies is commonly more dependent on the locations and values of the high and low anomaly peaks, where the approximation above is fully valid. To illustrate this, I show a simple synthetic model.

Figure D-1 shows an example of a magnetic anomaly caused by a point dipole source (equivalent to uniformly magnetized sphere) as observed on a horizontal plane. The source dipole, with a magnetic moment of 8×10^9 Am² (8 (A/m) $\times 1$ km³) in the direction of 45° inclination and -7° declination, parallel to the ambient magnetic field direction, is situated at the centre at a depth of 1000 m below the observation plane. The left panel shows the PTA anomaly (~1400 nT p-p) at the contour interval of 50 nT, while the thin contours in the right panel are for the TIA anomaly. It will be almost impossible to discriminate between the two in the illustration, but there is an actual difference as shown by the thick broken contour lines (at a contour interval of 2 nT) on the right panel. The peak value of the difference ε_T is ~14 nT, although the same difference might arise from a horizontal position inaccuracy of 6 m. If we use this TIA data in an inversion analysis for PTA anomalies, how much error would be obtained in the result?

The *PTA* and *TIA* data in the range of thick rectangle in Figure D-1 were put into inversion analyses for a single point dipole source (with six unknown parameters, three for position and three for magnetic moment) assuming input data are *PTA* anomalies. Table D-1 is the result for two source intensities, for the case shown in Figure D-1 and for the case of a smaller ($\times 0.3$) magnetic moment. The result using *PTA* data recovered the



Fig. D-1. Synthetic model example showing the difference between *PTA* (projected total intensity anomaly) and *TIA* (total intensity anomaly). (1) *PTA* at a contour interval of 50 nT, (2) Thin contours: *TIA* at a contour interval of 50 nT; Thick broken contours: the difference (*TIA* – *PTA*) at a contour interval of 2 nT. A point source model at the centre with a magnetic moment of 8×10^9 Am² in the direction of 45° inclination and -7° declination is assumed at the depth of 1000 m below the observation plane. The data within the range of thick rectangle are put into inversion analyses (assuming input data are *PTA* anomalies) to give the result in Table D-1.

 Table D-1.
 Results of inversion analyses by single dipole source model,

 for the cases of *PTA* and *TIA* data described in Fig. D-1 and for the cases of smaller (×0.3) source magnetic moment.

Parameters	Same model as in Fig. D-1		Case of \times 0.3 mag. moment		
					Result from <i>PTA</i>
	Source location				
	Northing (m)	1000.00	1001.99	1000.00	1000.60
Easting (m)	1000.00	999.81	1000.00	999.94	
Depth (m)	1000.00	999.91	1000.00	999.96	
Magnetic moment					
Intensity (Am ²)	7.99999	8.01194	2.40000	2.40102	
Inclination (°)	44.9999	45.3433	44.9999	45.1036	
Declination (°)	-7.0000	-6.9958	-6.9999	-6.9993	

source parameters perfectly, because no error or noise components are included. The result using *TIA* data also recovered source parameters very well, in spite of the approximation TIA = PTA. The source location could be recovered with an error in location of 2.0 m against the 2000 m range of analysis, and the error in magnetic moment was 0.15% in magnitude and 0.34° in direction. Here it is noted that the error becomes smaller, when the anomaly amplitude decreases, more rapidly than proportional to the source anomaly intensity.

The validity of approximating observed *TIA* anomalies with *PTA* anomalies is mentioned in most textbooks (e.g. Blakely, 1995, p. 179; SEGJ, 1999, p. 483). As this approximation is common among exploration geophysicists concerned with mobile magnetic surveys, the description of the effect of the approximation is usually omitted.

Isezaki and Matsuo presented an example of three-component magnetic survey data of Aogashima in their figure 7. Even in the colour scale with steps every 500 nT, there are repeated patterns of fluctuation along survey lines, with wavelengths of 0.5-1 km. I suppose the amplitudes of such fluctuation to be much larger than 100 nT p-p, and that this is the noise level (probably coming from attitude data) of their three-component measurement in Aogashima. Such fluctuations could not be reflected in the 3D inversion results (figures 9–11), and the misfit in the inversion process (figure 8) remained over 200 nT when expressed as a standard deviation, which is probably consistent with the values 2.5–3.5 for the 'goodness-of-fit ratio'.

If the vector magnetic anomaly observation can be performed in practice only with system errors of more than 100 nT, or if the

3D inversion analysis of the vector magnetic anomaly can do no better than a misfit of 200 nT, then the peak difference (14 nT in my example above) between *PTA* and *TIA* will be of little importance. In my opinion, the development of the threecomponent magnetic survey system is actually one of the most important topics to report, but the high-resolution ability of total intensity measurements (owing to the developments of proton precession and optical pumping magnetometers) is still a powerful benchmark even though the *TIA* is not a harmonic field component in the strict sense.

In addition to the discussion above, I draw attention to some further points to be clarified or corrected.

- 1) In figure 5, the result of model inversion of a kind of magnetization mapping is given, under the condition that the case of (*c*) *TIA* only includes an approximation in principle. But the description of the magnetization contradicts the grey-scale legend.
- 2) The complete recovery of source structure in (b) PTA and (d) three-component anomaly means simply that the simultaneous linear equations with no errors could be solved correctly. If another configuration were selected so that the complete fitting could not be attained (e.g. if the inversion model cells were shifted half a grid-spacing from the actual source model, or if model data from a spherical or ellipsoidal source were treated), the result of analysis might be more informative.
- 3) In figure 9, we see a strange saw-tooth feature in the observed data and calculated anomalies, which should be explained.
- 4) As is well known, a magnetic anomaly distribution cannot give a unique solution for its causative source distribution. Inversion of the magnetic anomaly then requires further constraints to restrict the resulting source distribution and properties, and the strategic concept used to overcome this difficulty should be given.

References

- Blakely, R. J., 1995, Potential theory in gravity and magnetic applications: Cambridge University Press, 441p.
- Society of Exploration Geophysicists of Japan, 1999, Butsuri-Tansa handbook Methodology: SEGJ, 944p (in Japanese).

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